PERGAMON

International Journal of Heat and Mass Transfer 46 (2003) 643–651

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Thermally developing forced convection in a porous medium: parallel plate channel with walls at uniform temperature, with axial conduction and viscous dissipation effects

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Received 6 May 2002; received in revised form 3 August 2002

Abstract

A modified Graetz methodology is applied to investigate the thermal development of forced convection in a parallel plate channel filled by a saturated porous medium, with walls held at uniform temperature, and with the effects of axial conduction and viscous dissipation included. The Brinkman model is employed. The analysis leads to expressions for the local Nusselt number, as a function of the dimensionless longitudinal coordinate and other parameters (Darcy number, Péclet number, Brinkman number).

1. Introduction

Because of the use of hyperporous media in the cooling of electronic equipment, there has recently been renewed interest in the problem of forced convection in a porous medium channel. In their recent survey of the literature, Nield and Bejan [1] refer to over 30 papers on this topic, but none of them deals explicitly with the case of thermal development. This gap in the literature has been partly filled by Nield et al. [2–4]. In each of these papers it was assumed that the Péclet number was sufficiently large so that longitudinal (axial) conduction could be neglected. This approximation allows the use of analysis close to the classical Graetz analysis.

In the present paper the more general case, in which longitudinal conduction is significant, is treated. The Graetz analysis in an extended form, as discussed by Lahjomri et al. [5,6], is followed here. (In [5] are listed 10 analytical papers (and six numerical studies) on the extended Graetz problem in which a variety of approaches have been used; we consider the approach used in [5] to be the most satisfactory analytical approach presented to date.) We treat the case of a channel confined by parallel plane walls at which the temperature is held piecewise constant, with a step at the entrance section. For the case of the Darcy model, the hydrodynamically developed velocity profile is that of slug flow, and the problem is particularly simple, but in this paper we consider the more complicated flow appropriate to the Brinkman model.

The incorporation of the effect of axial conduction requires a major change in approach (because of the upstream propagation of temperature changes). However, once this change has been made it is comparatively simple to incorporate also the effect of viscous dissipation. This second change results in a homogeneous differential equation problem being replaced by a non-homogeneous one.

Previous work on the effects of axial conduction and viscous dissipation in ducts, in the case of fluids clear of

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0017-9310/03/$ - see front matter
PII: S0017-9310(02)00327-7

Nomenclature

A_n,B_n	coefficients defined by Eqs. (24a,b)	T^*_{IN}	inlet wall temperature
Br	Brinkman number, defined by Eq. (15)	T^*_m	bulk mean temperature
c_p	specific heat at constant pressure	T^*_w	downstream wall temperature
d_0, d_1, d_2, d_3	constants defined by Eqs. ((30a)–(d))	u	$\mu u^*/GH^2$
D	function defined by Eqs. (16a)–(16c)	u^*	filtration velocity
Da	Darcy number, K/H^2	$\hat{u}$	u^*/U^*
f_n	eigenfunctions for the upstream region	U^*	mean velocity
g_n	eigenfunctions for the downstream region	x^*	longitudinal coordinate
F	function defined by Eqs. (20a)–(20c)	y^*	transverse coordinate
G	applied pressure gradient		
H	half channel width	*Greek symbols*	
k	fluid thermal conductivity	β_n	eigenvalues for downstream region
K	permeability	ξ	x^*/PeH
M	μ_{eff}/μ	η	y^*/H
Nu	local Nusselt number defined by Eq. (11)	θ	$(T^* - T^*_w)/(T^*_{IN} - T^*_w)$
Pe	Péclet number defined by Eq. (3)	λ_n	eigenvalues for the upstream region
q''	wall heat flux	μ	fluid viscosity
S	$(M\,Da)^{-1/2}$	μ_{eff}	effective viscosity in the Brinkman term
T^*	temperature	ρ	fluid density

solid material, has been surveyed by Shah and London [7].

2. Analysis

2.1. Basic equations

For the steady-state hydrodynamically developed situation we have unidirectional flow in the x^*-direction between impermeable boundaries at $y^* = -H$ and $y^* = H$, as illustrated in Fig. 1. For $x^* > 0$ the (downstream) temperature on each boundary is held constant at the value T^*_w. For $x^* < 0$ the inlet (upsteam) wall temperature T^*_{IN} is assumed constant on each boundary.

The Brinkman momentum equation is

$$\mu_{eff}\frac{d^2u^*}{dy^{*2}} - \frac{\mu}{K}u^* + G = 0, \tag{1}$$

where μ_{eff} is an effective viscosity, μ is the fluid viscosity, K is the permeability, and G is the applied pressure gradient.

We define dimensionless variables

$$\xi = \frac{x^*}{PeH}, \quad \eta = \frac{y^*}{H}, \quad u = \frac{\mu u^*}{GH^2}. \tag{2}$$

Here the Péclet number Pe is defined by

$$Pe = \frac{\rho c_p H U^*}{k}. \tag{3}$$

The dimensionless form of Eq. (1) is

$$M\frac{d^2u}{d\eta^2} - \frac{u}{Da} + 1 = 0. \tag{4}$$

We have defined the viscosity ratio M and the Darcy number Da by

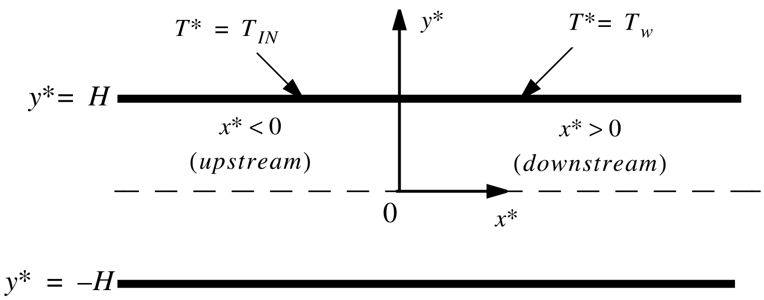

Fig. 1. Definition sketch.

$$M = \frac{\mu_{\mathrm{eff}}}{\mu}, \quad Da = \frac{K}{H^2}. \tag{5}$$

The solution of this equation subject to the boundary condition $u = 0$ at $\eta = 1$, and the symmetry condition $\mathrm{d}u/\mathrm{d}\eta = 0$ at $\eta = 0$ is

$$u = Da\left(1 - \frac{\cosh S\eta}{\cosh S}\right), \tag{6}$$

where

$$S = \left(\frac{1}{MDa}\right)^{1/2}. \tag{7}$$

This parameter is introduced for convenience. It will be noted that, from now on in the analysis, M and Da appear only in the combination of M times Da, so that without loss of generality one can assume that $M = 1$ in the presentation of results.

The mean velocity U^* and the bulk mean temperature T_{m}^* are defined by

$$U^* = \frac{1}{H}\int_0^H u^*\,\mathrm{d}y^*, \quad T_{\mathrm{m}}^* = \frac{1}{HU^*}\int_0^H u^* T^*\,\mathrm{d}y^*. \tag{8}$$

Further dimensionless variables are defined by

$$\hat{u} = \frac{u^*}{U^*}, \quad \theta = \frac{T^* - T_{\mathrm{w}}^*}{T_{\mathrm{IN}}^* - T_{\mathrm{w}}^*}. \tag{9}$$

This implies that

$$\hat{u} = \frac{S\cosh S - S\cosh S\eta}{S\cosh S - \sinh S}. \tag{10}$$

The Nusselt number Nu is defined as

$$Nu = \frac{2Hq''}{k(T_{\mathrm{w}}^* - T_{\mathrm{m}}^*)}. \tag{11}$$

(The reader should note that we have followed Nield and Bejan [1] and defined Nu in terms of the channel width rather than the hydraulic diameter. The Nusselt number defined in terms of the hydraulic diameter is twice Nu.)

Local thermal equilibrium is assumed. The steady-state thermal energy equation is then

$$\rho c_p u^* \frac{\partial T^*}{\partial x^*} = k\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right) + \Phi, \tag{12}$$

where Φ is the contribution due to viscous dissipation. The modeling of this viscous term is controversial. The simplest expression, which is appropriate to the Darcy equation, in the present case is

$$\Phi = \frac{\mu u^{*2}}{K}. \tag{13a}$$

Nield [8] argued that the viscous dissipation should remain equal to the power of the drag force when the Brinkman equation is considered, and in the present case this implies that

$$\Phi = \frac{\mu u^{*2}}{K} - \mu_{\mathrm{eff}} u^* \frac{\mathrm{d}^2 u^*}{\mathrm{d}y^{*2}}. \tag{13b}$$

On the other hand, Al-Hadhrami et al. [9] proposed a form which is compatible with an expression derived from the Navier-Stokes equation for a fluid clear of solid material, in the case of large Darcy number, and in this case we have

$$\Phi = \frac{\mu u^{*2}}{K} + \mu\left(\frac{\mathrm{d}u^*}{\mathrm{d}y^*}\right)^2. \tag{13c}$$

In each case the added Brinkman term is $\mathrm{O}(Da)$ in comparison with the Darcy term. Consequently, in the case of small Da the three models are effectively equivalent to each other.

In non-dimensional form this becomes

$$\hat{u}\frac{\partial\theta}{\partial\xi} = \frac{1}{Pe^2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial\eta^2} + BrD(S,\eta), \tag{14}$$

where the Brinkman number Br is defined as

$$Br = \frac{\mu U^{*2} H^2}{k(T_{\mathrm{IN}}^* - T_{\mathrm{w}}^*)K}, \tag{15}$$

and, corresponding to the three models, we have the alternative expressions

$$D(S,\eta) = \left[\frac{S\cosh S - S\cosh S\eta}{S\cosh S - \sinh S}\right]^2, \tag{16a}$$

$$D(S,\eta) = \frac{S^2\cosh S(\cosh S - \cosh S\eta)}{(S\cosh S - \sinh S)^2}, \tag{16b}$$

$$D(S,\eta) = \frac{S^2(\cosh^2 S - 2\cosh S\cosh S\eta + \cosh 2S\eta)}{(S\cosh S - \sinh S)^2}. \tag{16c}$$

2.2. Extended Graetz analysis

The problem now is to solve Eq. (14) subject to the conditions

$$\theta_1 = 1 \quad \text{at } \eta = 1 \quad \text{for } \xi < 0,$$
$$\theta_2 = 0 \quad \text{at } \eta = 1 \quad \text{for } \xi > 0,$$
$$\frac{\partial\theta_i}{\partial\eta} = 0 \quad \text{at } \eta = 0 \quad \text{for all } \xi, (i = 1,2),$$
$$\theta_1 = \theta_2 \quad \text{at } \xi = 0 \quad \text{for } 0 < \eta < 1,$$
$$\frac{\partial\theta_1}{\partial\xi} = \frac{\partial\theta_2}{\partial\xi} \quad \text{at } \xi = 0 \quad \text{for } 0 < \eta < 1. \tag{17a, b, c, d, e}$$

Eqs. (17d) and (17e) express the continuities of the temperature and the heat flux at the entrance section $\xi = 0$. For infinitely large values of $|\xi|$, the dimensionless temperature is the particular solution of the equation

$$\frac{\partial^2 \theta_i}{\partial \eta^2} = -BrD(S, \eta). \tag{18}$$

Following Lahjomri et al. [5,6], we use a separation of variables method to generate the general solution of Eq. (14) in the upstream and downstream regions satisfying the conditions (17a,b,c) and (18). This solution can be represented by

$$\theta_1(\xi, \eta) = 1 + \sum_{n=1}^{\infty} A_n f_n(\eta) \exp(\lambda_n^2 \xi) + BrF(S, \eta) \quad \text{for } \xi < 0,$$
$$\theta_2(\xi, \eta) = \sum_{n=1}^{\infty} B_n g_n(\eta) \exp(-\beta_n^2 \xi) + BrF(S, \eta) \quad \text{for } \xi > 0, \tag{19a, b}$$

where, corresponding to the three models for viscous dissipation,

$$F(S, \eta) = \frac{\frac{1}{4}S^2(1 + 2\cosh^2 S)(1 - \eta^2) + 2\cosh S(\cosh S\eta - \cosh S) - \frac{1}{8}(\cosh 2S\eta - \cosh 2S)}{(S\cosh S - \sinh S)^2}, \tag{20a}$$

$$F(S, \eta) = \frac{\frac{1}{2}S^2 \cosh^2 S(1 - \eta^2) - \cosh S(\cosh S - \cosh S\eta)}{(S\cosh S - \sinh S)^2}, \tag{20b}$$

$$F(S, \eta) = \frac{\frac{1}{2}S^2 \cosh^2 S(1 - \eta^2) + 2\cosh S(\cosh S\eta - \cosh S) - \frac{1}{4}(\cosh 2S\eta - \cosh 2S)}{(S\cosh S - \sinh S)^2}. \tag{20c}$$

The λ_n and β_n are eigenvalues associated with the eigenfunctions f_n and g_n, respectively, and the A_n and B_n are coefficients to be determined from the matching condition (17d,e) (see below). The eigenfunctions f_n and g_n are the solutions of the following differential equations:

$$\frac{d^2 f_n}{d\eta^2} + \lambda_n^2 \left[\frac{\lambda_n^2}{Pe^2} - \hat{u}(\eta)\right] f_n = 0,$$
$$\frac{d^2 g_n}{d\eta^2} + \beta_n^2 \left[\frac{\beta_n^2}{Pe^2} + \hat{u}(\eta)\right] g_n = 0 \tag{21a, b}$$

satisfying the boundary conditions

$$f_n'(0) = 0 \quad \text{and} \quad f_n(1) = 0,$$
$$g_n'(0) = 0 \quad \text{and} \quad g_n(1) = 0. \tag{22a, b}$$

From the matching conditions (17d,e), we obtain the following equations determining the coefficients A_n and B_n:

$$1 + \sum_{n=1}^{\infty} A_n f_n(\eta) = \sum_{n=1}^{\infty} B_n g_n(\eta),$$
$$\sum_{n=1}^{\infty} \lambda_n^2 A_n f_n(\eta) = -\sum_{n=1}^{\infty} \beta_n^2 B_n g_n(\eta). \tag{23a, b}$$

The eigenvalue problem constituted by Eqs. (21a,b) and (22a,b) is not of the classical Sturm-Liouville type and so the usual orthogonality formula is not valid. However, as Lahjomri et al. [5,6] showed, the coefficients can still be isolated from each other, and are given by the formulas

$$A_n = \frac{-\int_0^1 \left[\frac{\lambda_n^2}{Pe^2} - \hat{u}(\eta)\right] f_n \, d\eta}{\int_0^1 \left[\frac{2\lambda_n^2}{Pe^2} - \hat{u}(\eta)\right] f_n^2 \, d\eta},$$
$$B_n = \frac{\int_0^1 \left[\frac{\beta_n^2}{Pe^2} + \hat{u}(\eta)\right] g_n \, d\eta}{\int_0^1 \left[\frac{2\beta_n^2}{Pe^2} + \hat{u}(\eta)\right] g_n^2 \, d\eta}. \tag{24a, b}$$

For large values of the Péclet number ($Pe \to \infty$) and when $S = 0$ and $Br = 0$, the solution tends to the classical Graetz problem without axial conduction, and one finds that $\theta_1(\xi, \eta)$ tends to 1 (a uniform temperature profile in the upstream region), as expected.

The dimensionless bulk temperature $\theta_{b,i}(\xi)$ and the local Nusselt number $Nu_i(\xi, \eta)$ (based on the gap width $2H$ rather than the hydraulic diameter) for the upstream and downstream regions are given by

$$\theta_{b,i}(\xi) = \int_0^1 \hat{u}(\eta) \theta_i \, d\eta, \tag{25}$$

$$Nu_i = -\frac{2\left[\frac{\partial \theta_i}{\partial \eta}\right]_{\eta=1}}{\theta_{b,i} - [\theta_i]_{\eta=1}} \quad (i = 1, 2). \tag{26}$$

In particular, from Eqs. (25), (26), (19b) and (21b), the local Nusselt number for the downstream region ($\xi > 0$) is given by

$$Nu_2(\xi) = \frac{2\sum_{n=1}^{\infty} B_n g_n'(1)\exp(-\beta_n^2\xi) + 2BrF'(S,1)}{\sum_{n=1}^{\infty} B_n \exp(-\beta_n^2\xi)\left[(g_n'(1)/\beta_n^2) + (\beta_n^2/Pe^2)\int_0^1 g_n(\eta)\mathrm{d}\eta\right] - Br\int_0^1 \hat{u}(\eta)F(S,\eta)\mathrm{d}\eta}. \quad (27)$$

The terms multiplied by Br in Eq. (27) can be evaluated. For example, when $F(S,\eta)$ is given by Eq. (20a), one has

$$F'(S,1) = \frac{3S\sinh S\cosh S - S^2 - 2S^2\cosh^2 S}{2(S\cosh S - \sinh S)^2}, \quad (28)$$

$$\begin{aligned}\int_0^1 \hat{u}(\eta)F(S,\eta)\mathrm{d}\eta &= \frac{S}{S\cosh S - \sinh S}\left\{d_0\left(\cosh S - \frac{\sinh S}{S}\right)\right. \\ &\quad + d_1\left[\frac{\cosh S}{3} - \frac{(S^2+2)\sinh S}{S^3} + \frac{2\cosh S}{S^2}\right] \\ &\quad + d_2\left(\frac{\sinh 2S}{4S} - \frac{1}{2}\right) \\ &\quad \left. + d_3\left(\frac{\sinh 2S\cosh S}{2S} - \frac{\sinh S}{2S} - \frac{\sinh 3S}{6S}\right)\right\} \quad (29)\end{aligned}$$

where the constants d_0, d_1, d_2, d_3 are defined by

$$\begin{aligned} d_0 &= \frac{2S^2 - 1 + (4S^2 - 14)\cosh^2 S}{8(S\cosh S - \sinh S)^2}, \\ d_1 &= -\frac{S^2 + 2S^2\cosh^2 S}{4(S\cosh S - \sinh S)^2}, \\ d_2 &= \frac{2\cosh S}{(S\cosh S - \sinh S)^2}, \\ d_3 &= -\frac{1}{8(S\cosh S - \sinh S)^2}. \end{aligned} \quad (30\mathrm{a,b,c,d})$$

2.3. Calculation procedure

At this stage of their analysis, Lahjomri et al. [5] use a coordinate transformation to transform the differential Eqs. (21a,b) into a standard form of Mathieu's modified differential equation, but we prefer a more direct approach, namely using a shooting method to obtain the eigenvalues and the corresponding eigenfunctions simultaneously. (One advantage is that our method can deal with arbitrary velocity profiles, and not just hyperbolic–cosine ones.) We express the second order Eq. (21a) as a system of two first order ones, by writing $y_1 = f_n, y_2 = f_n'$, where a prime denotes a derivative with respect to η. Then

$$\begin{aligned} y_1' &= y_2, \\ y_2' &= \lambda_n^2\left[\hat{u} - \frac{\lambda_n^2}{Pe^2}\right]y_1. \end{aligned} \quad (31\mathrm{a,b})$$

These equations may be solved by a shooting procedure. Each eigenfunction may be normalized by the requirement that it satisfies the condition $f_n(0) = 1$. Then we have

$$y_1(0) = 1, \quad y_2(0) = 0. \quad (32\mathrm{a,b})$$

Starting with an estimate for the value of the nth eigenvalue λ_n, one can step forward from $\eta = 0$ to 1 and vary the value of λ_n to satisfy the condition $y_1(1) = 0$. This yields the precise eigenvalue, and the corresponding function $y_1(x)$ is the required eigenfunction $f_n(\eta)$. We recognize that $f_n'(1)$ is just $y_2(1)$. In a similar fashion one can calculate β_n and $g_n(\eta)$. Once the eigenvalues and eigenfunctions have been obtained, the coefficients A_n and B_n can be obtained by simple numerical integration of the integrals that are involved, and the solution is readily completed. (In fact, for our bulk computations we found it convenient to compute Nu directly from Eqs. (19a,b), (21a,b) and (26) and to just use Eq. (27) as a check.) We checked that the shooting procedure lead to accurate results for the eigenvalues by checking special limiting cases, such as the standard Graetz problem. We checked the numerical convergence of our series as we went along. An indication of the number of modes required for convergence is given in Fig. 2 of [5]. As expected, that number of modes increases as ξ becomes small. There is a singularity at $\xi = 0$ that is dealt with in the classical Graetz problem by obtaining a special asymptotic solution (the Levêque solution). We found that we could handle several hundred modes, and thereby calculate Nu fairly accurately for quite small values of ξ. We therefore judged that the effort of finding a special asymptotic solution was not warranted.

3. Results and discussion

In this problem there are a large number of parameters to vary, and the calculations are time consuming. We have the ability to calculate the temperature field throughout the flow region, but in the interests of brevity we will just present values of the Nusselt number, and those for the downstream flow region only. (We quickly see from a simple asymptotic analysis based on Eq. (14) that, as a result of the inclusion of the axial conduction term, the temperature changes will be felt upstream a distance ξ of order Pe^{-2}.)

First we consider the case in which viscous dissipation is negligible ($Br = 0$). Plots of the downstream Nusselt number are presented in Figs. 2 and 3. Fig. 2 is for the case $Da = 10^{-5}$, which is the smallest Darcy number for which we could make the calculations and which approximates the case of Darcy (slug) flow. Fig. 3

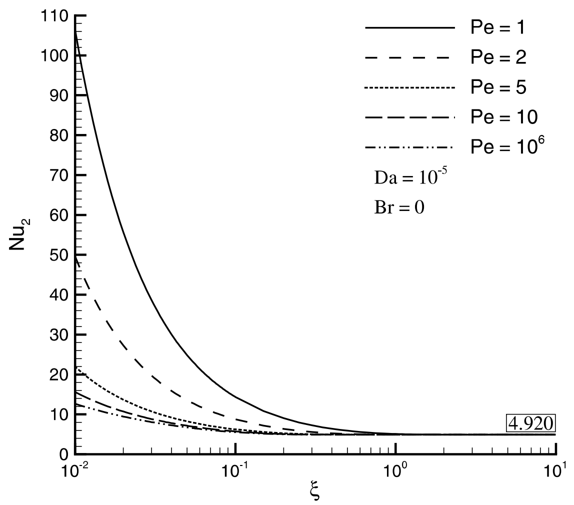

Fig. 2. Plots of downstream local Nusselt number versus dimensionless axial coordinate, for the case of negligible viscous dissipation and for small Darcy number, for various values of the Péclet number.

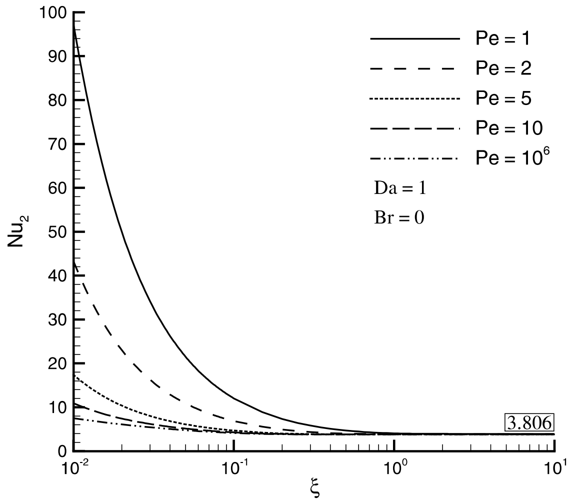

Fig. 3. Plots of downstream local Nusselt number versus dimensionless axial coordinate, for the case of negligible viscous dissipation and for large Darcy number, for various values of the Péclet number.

is for the case $Da = 1$, which is representative of a hyperporous medium. It is clear that an increase in Da results in an increase of the thermally developing Nusselt number by a comparatively small amount. (The increase is not surprising, since one would expect that a less restrictive medium would lead to greater convection.) The Nusselt number for large ξ is the fully developed value. The value 4.920 for the case $Da = 10^{-5}$ is close to the known value 4.935 ($\pi^2/2$) for the Darcy flow (slug flow) limit. The value 3.806 for the case $Da = 1$ is close to the known value 3.770 for the plane Poisueille flow limit.

In contrast, the developing Nusselt number is strongly dependent on the value of the Péclet number Pe. The case of large Pe number ($Pe = 10^6$) illustrates the situation where the axial conduction term is negligible. As one would expect, our results for this case agree with results based on our previous analysis. In each of Figs. 2 and 3 the plot for $Pe = 10$ is not far from that for $Pe = 10^6$, but for smaller values of Pe the increase in the value of the developing Nu (for a fixed value of ξ) is quite dramatic, the value varying with $1/Pe$ approximately.

We now move on to consider the effect of viscous dissipation. Figs. 4 and 5 are for the case of very large Pe, where the effect of axial conduction is negligible (and again for the small Da and large Da cases, respectively). A feature of considerable interest is that even a small amount of viscous dissipation (non-zero Br) leads to a jump in the fully developed Nu_2 to a value which is then independent of Br, and this effect is especially noticeable in the case of small Darcy number. (The jump is not too surprising when one observes that the change from zero Br to non-zero Br changes Eq. (14) from a homogeneous equation into a non-homogeneous equation, and this is analogous to changing a free oscillation problem into a forced oscillation problem. Viscous dissipation provides a heat source distribution which persists downstream (unlike the heat flux at walls subject to a constant-temperature boundary condition, which decays downstream) and changes the nature of the fully developed temperature distribution.) We also see a dramatic difference between the effect of positive Br and the effect of

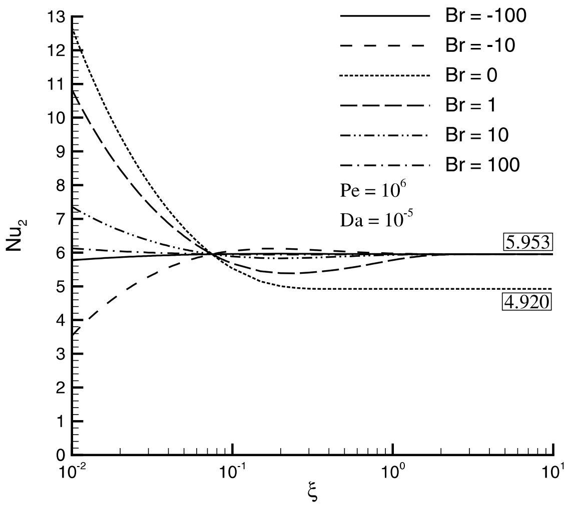

Fig. 4. Plots of downstream local Nusselt number versus dimensionless axial coordinate, for the case of negligible axial conduction (large Péclet number) and for small Darcy number, for various values of the Brinkman number.

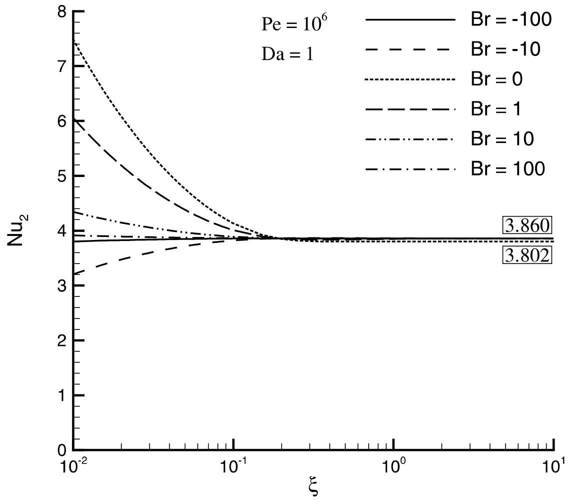

Fig. 5. Plots of downstream local Nusselt number versus dimensionless axial coordinate, for the case of negligible axial conduction (large Péclet number) and for large Darcy number, for various values of the Brinkman number.

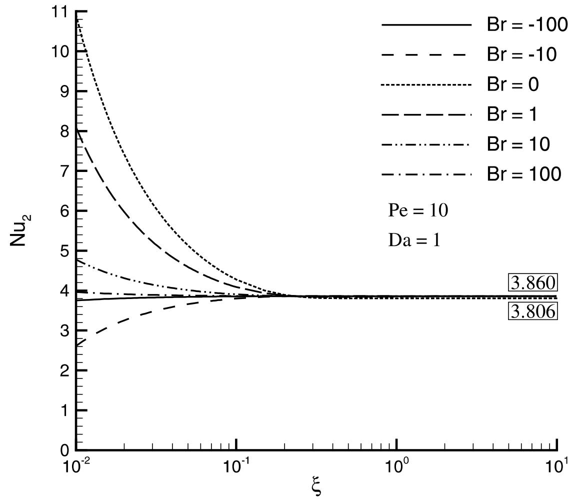

Fig. 7. As for Fig. 5, but with $Pe = 10$.

negative Br. The case $Br > 0$ corresponds to incoming fluid being heated at the walls. The viscous dissipation produces a (generally non-uniform) distribution of positive heat sources, and this reinforces the heating effect as the fluid moves downstream. As ξ increases the value of the Nusselt number passes through a minimum. For very large values of Br the value of Nu changes only slowly with ξ. The case $Br < 0$ corresponds to incoming fluid being cooled at the walls, and this cooling at the walls is opposed by the heating due to viscous dissipation in the bulk of the fluid. This opposition is particularly dramatic for the case $Br = -1$, for which the difference between the wall temperature and the bulk temperature changes sign at some value of ξ. That means that the Nusselt number based on that difference becomes quantitatively meaningless, and for that reason we have not plotted in our figures any curve for that value of Br. For $Br = -10$ or less, the plots for Nu_2 are regular and exhibit a maximum value at some value of ξ.

In Figs. 6 and 7, and again in Figs. 8 and 9 we present companions to Figs. 4 and 5, now for the cases of $Pe = 10$ and $Pe = 1$, respectively. Figs. 4 and 5 as a pair do not differ much from Figs. 6 and 7, and this drives home the point that when $Pe = 10$ the effect of axial conduction is not dramatically significant. When $Pe = 1$, the effect of axial conduction is more dramatic. It results in an increase in the variation of Nu_2 as the flow develops. In particular, it results in Nu_2 becoming negative for small

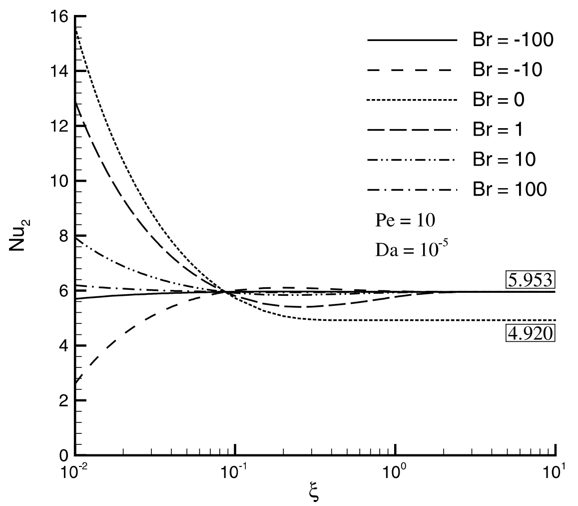

Fig. 6. As for Fig. 4, but with $Pe = 10$.

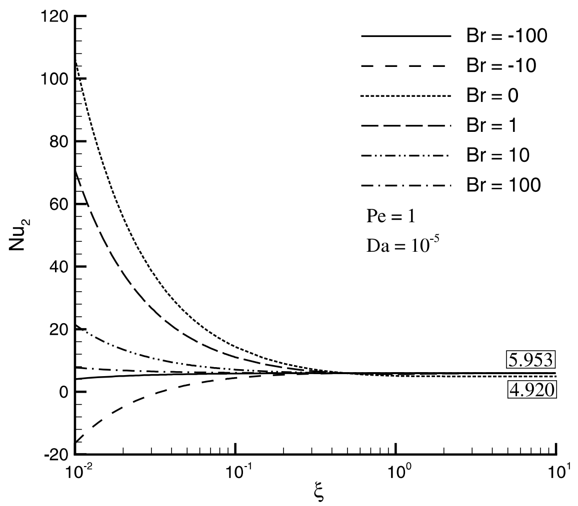

Fig. 8. As for Fig. 4, but with $Pe = 1$.

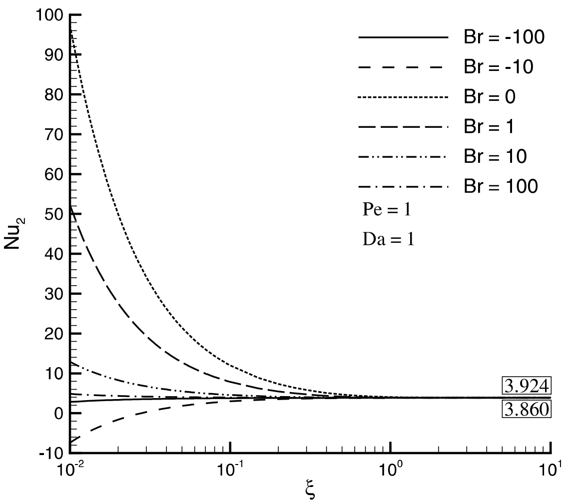

Fig. 9. As for Fig. 5, but with $Pe = 1$.

values of ξ when *Br* is moderately large and negative. In the circumstances of Fig. 9 ($Pe = 1, Da = 10$) the jump in the value of the fully developed Nusselt number as *Br* goes from zero to a non-zero value is very small.

In Fig. 10 (together with Fig. 5) we present a comparison of results based on the various models of viscous dissipation, for a case in which the Darcy number is large, and for which the effect of axial conduction is negligible. The most dramatic difference between the models is in the value of the fully developed Nusselt number. This number has the values 3.860, 4.160, 6.641, for the respective models.

The analysis used in this paper has an important limitation. The ansatz assumed in writing down Eq. (19a,b) implies that temperature at a great distance downstream is independent of the axial coordinate. This assumption is a sensible one for a discussion of thermally developing flow. It is also a sensible assumption to apply at the exit cross-section when using numerical modeling. However, it is not a good assumption when considering the limit as the thermal convection becomes fully developed. In fact, it violates the First Law of Thermodynamics when the viscous dissipation is not zero. Thus the jump in the value of the fully developed Nusselt number as *Br* goes from zero to a non-zero value should be regarded as an artifact of the mathematical modeling. Likewise, not much should be read into the fact that the fully developed Nusselt number for non-zero *Br* is independent of *Pe* (compare Figs. 4, 6, 8 and also Figs. 5, 7, 9).

4. Conclusions

We have investigated the effect of adding an axial conduction term and a viscous dissipation term to the thermal energy equation for the problem of forced convection in a parallel-plate channel, with the temperature held constant at the walls. In the absence of viscous dissipation, the developing Nusselt number varies little with the Darcy number but quite dramatically with the Péclet number (increasing as *Pe* decreases). The effect of viscous dissipation has a significant effect on the developing Nusselt number.

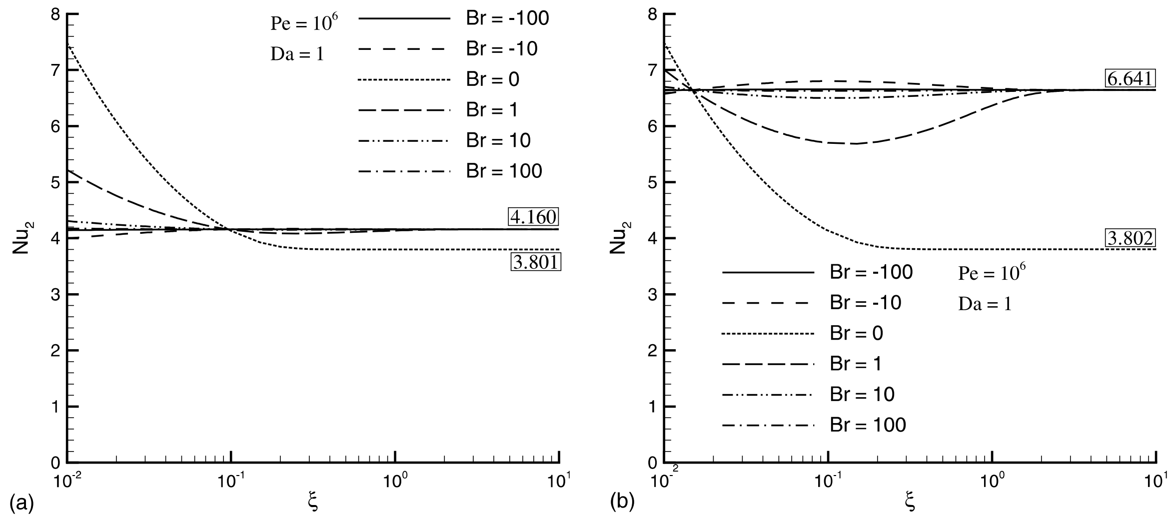

Fig. 10. Comparison of different models for viscous dissipation: (a) model based on power of drag force (Nield [8]), (b) model based on compatibility with a clear fluid (Al-Hadhrami et al. [9]). Other parameters are as in Fig. 5, which is based on a model in which the contribution of the Brinkman term to the viscous dissipation is ignored in comparison to the contribution of the Darcy term.

Acknowledgements

A.V.K. would like to acknowledge grant #01/ER-12 awarded to him by the United Engineering Foundation and the support provided by the North Carolina Supercomputing Center (NCSC) under the Advanced Computing Resources Grant.

References

[1] D.A. Nield, A. Bejan, Convection in Porous Media, second ed., Springer-Verlag, New York, 1999.
[2] D.A. Nield, A.V. Kuznetsov, M. Xiong, Thermally developing forced convection in a porous medium: parallel plate channel or circular tube with walls at constant temperature, J. Porous Media, in press.
[3] D.A. Nield, A.V. Kuznetsov, M. Xiong, Thermally developing forced convection in a porous medium: parallel plate channel or circular tube with walls at constant heat flux, J. Porous Media, in press.
[4] D.A. Nield, A.V. Kuznetsov, M. Xiong, Effect of local thermal non-equilibrium on thermally developing forced convection in a porous medium, Int. J. Heat Mass Transfer, in press.
[5] J. Lahjomri, A. Oubarra, A. Alemany, Heat transfer by laminar Hartmann flow in thermal entrance region with a step change in wall temperature: the Graetz problem extended, Int. J. Heat Mass Transfer 45 (2002) 1127–1148.
[6] J. Lahjomri, A. Oubarra, Analytical solution of the Graetz problem with axial conduction, ASME J. Heat Transfer 121 (1999) 1078–1083.
[7] R.K. Shah, A.L. London, Laminar flow forced convection in ducts, Adv. Heat Transfer (Suppl. 1) (1978) 109–118 and 183–184.
[8] D.A. Nield, Resolution of a paradox involving viscous dissipation and nonlinear drag in a porous medium, Transport Porous Media 41 (2000) 349–357.
[9] A.K. Al-Hadhrami, L. Elliott, D.B. Ingham, A new model for viscous dissipation in porous media across a range of permeability values, Transport Porous Media, submitted for publication.